Optimal Sampling for State Change Detection with Application to the Control of Sleep Mode

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Outline

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1. IEEE 802.16e Power Save Mode
2. System Model
3. Parametric Optimization
4. Dynamic Programming
5. Suboptimal Policies
6. Numerical Results
7. Conclusion

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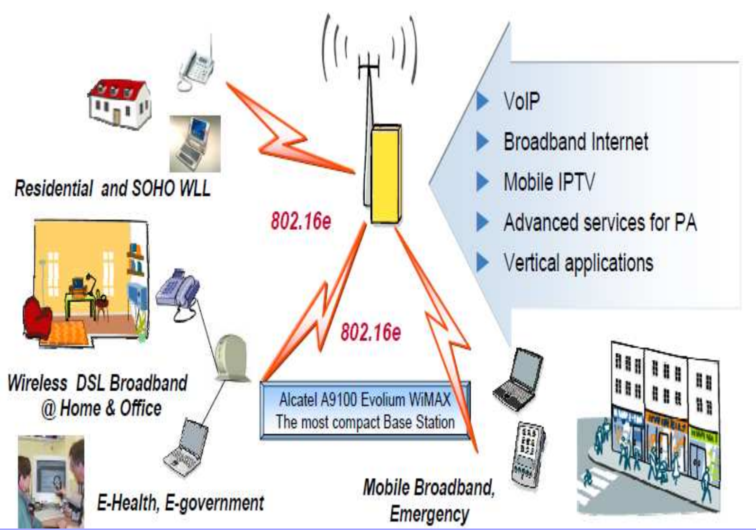
WiMAX: Worldwide interoperable Microwave Acess standard



Power saving classes

- Type I
  - Best-Effort traffic
  - Non-Real Time Variable Rate traffic
  - Successive sleeping window
    - repeated window is twice the previous one (multiplicative).
- Type II
  - Unsolicited Grant Service traffic
  - Real Time Variable Rate traffic
  - Successive sleeping windows
    - all window have same size
- Type III
  - Multicast connections
  - Management operations
  - Only one sleeping window
    - window size is set to maximum value.

Objective

Questions

- Is IEEE 802.16e standard protocol optimal ?
- Why MULTIPLICATIVE increase ?
- Are random sleep window better ?
- Should we start with the lowest window size?

Some of these questions are answered in literature (including our previous work [1]) but restricted to *only poisson arrival process*.

In this work, we try to answer such questions for more *general arrival process*.

[1] S. Alouf and E. Altman and A. P. Azad, Analysis of an M/G/1 queue with repeated inhomogeneous vacations with application to IEEE 802.16e power saving mechanism, Proc. of Sigmetrics 2008

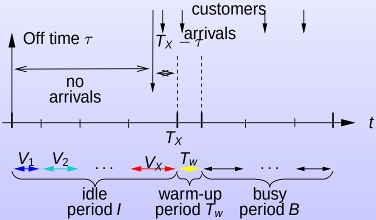
Objective

- More general modeling which can facilitate analytical study
  - beyond poisson arrival process
    - Exponentially distributed off time
    - Hyper-exponentially distributed off time
    - General distribution of off time
  - beyond WiMAX standard sleep duration
    - General deterministic sleep/vacation duration
    - Exponentially distributed sleep duration
- This model allows us to study the strategy which optimizes the energy saving and extra delay simultaneously.

Main Contribution

- We consider different strategies of power saving, and derive global optimal behaviour in different scenarios;
- We show that when the incoming traffic has a Poisson arrival process the optimal strategy is the repeated constant policy;
- For general traffic, we show that deterministic policies are optimal, and when the residual off time converges in distribution to some limit, then the optimal policy converges to a constant;
- We propose Suboptimal policies which performs better than parametric optimal but simpler than Optimal.
- Finally, the optimal performance is compared to the performance of the standards.

Off times



Hyper exponential distributed off times τ with n phases

$$f_\tau(t) = \sum_{i=1}^{n} q_i \lambda_i \exp(-\lambda_i t), \quad \sum_{i=1}^{n} q_i = 1.$$

Remark: $n = 1 \rightarrow$ Exponential distributed off times τ (Poisson Arrival)

Optimal Decision Model

- Consider a system with repeated vacation. It is needed to take decision at each vacation instant based on cost

$$V := \bar{\epsilon}\,\mathbb{E}[T_X - \tau] + \epsilon\,(E_L\mathbb{E}[X] + E_S\mathbb{E}[T_X]) \tag{1}$$

  - $T_X - \tau$ is the extra delay due to vacation.
  - $E_L\mathbb{E}[X] + E_S\mathbb{E}[T_X]$ is the energy consumption during vacation.
  - Weight factor $\epsilon \in [0, 1]$ balances the priority of system, more energy conscious or more delay conscious.
  - X is the number of vacations (random variable).
  - T_X is the Xth vacation completion time.
- Optimal decision parameters can be obtained by

$$\min_{\{B_k\}_{k\geq 1}} V \tag{2}$$

B_k is the distribution of kth vacation. (minimization over parameters of the distributions of B_k within a given class of distributions.)

Optimal Decision Model

We introduce the following dynamic programming formulation

$$V_k^\star = \min_{b_{k+1}} \left\{ \mathbb{E}[c(t_k, b_{k+1})] + P(\tau > t_k + b_{k+1} | \tau > t_k) V_{k+1}^\star \right\} \quad (3)$$

for $k \in \mathbb{N}$, where

$$c(t,b) := \bar{\epsilon}\,\mathbb{E}[(t+b-\tau)\mathbb{1}\{\tau \le t+b\}\,|\tau > t] + \epsilon(E_L + E_S b),$$

An equivalent total cost problem can be expresses as

$$\begin{aligned} V &= \sum_{k=1}^{\infty} \Big\{ \bar{\epsilon}\,\mathbb{E}[(T_k - \tau)\mathbb{1}\{T_{k-1} < \tau \le T_k\}] \\ &\quad + \epsilon P(X = k)\,(E_L k + E_S \mathbb{E}[T_k])\Big\}. \end{aligned} \quad (4)$$

Total Cost

Total cost for hyper exponential Off time τ

$$V = -\bar{\epsilon}\,\mathbb{E}[\tau] + \sum_{k=0}^{\infty}\sum_{i=1}^{n} q_i \mathcal{T}_k^*(\lambda_i)\left(\epsilon E_L + \eta \mathbb{E}[B_{k+1}]\right), \qquad (5)$$

where

$\mathbb{E}[\tau] = \sum_{i=1}^{n} q_i/\lambda_i$ is the expectation of τ,

$\eta = \bar{\epsilon} + \epsilon E_S$,

$T_k = \sum_{i=1}^{k} B_k$,

$\mathcal{T}_k^*(\lambda_i)$ is the Laplace transform of T_k,

$\{B_k\}_{k\in\mathbb{N}^*}$ is the generic random variable denoting kth vacation.

Parametric Optimization

Identically distributed vacations

Let B be a generic random variable denoting same distribution of all vacation duration.

$$V = -\bar{\epsilon}\,\mathbb{E}[\tau] + (\epsilon E_L + \eta \mathbb{E}[B]) \sum_{i=1}^{n} \frac{q_i}{1 - \mathcal{B}^*(\lambda_i)}. \tag{6}$$

Parametric Optimization

Minimize the total cost with some parameter of vacation duration

$$V^* = \min_{\{B\}} \left\{ -\bar{\epsilon}\,\mathbb{E}[\tau] + (\epsilon E_L + \eta \mathbb{E}[B]) \sum_{i=1}^{n} \frac{q_i}{1 - \mathcal{B}^*(\lambda_i)} \right\}. \tag{7}$$

Vacation Distributions

Strategies

- Exponentially distributed vacations; the parameter to optimize is the mean vacation size $b = \mathbb{E}[B]$;
- Equally sized vacations (periodic pattern); the parameter to optimize is the constant vacation b;
- General vacations that follow a scaled version of a known distribution; the parameter to optimize is the scale α;
- General discrete vacations; the parameter to optimize is the distribution $\mathbf{p}$.

Exponential B, Hyper-Exponential τ

The total cost

$$V_e(b) \quad = \quad \epsilon\left(E_S + \frac{E_L}{b}\right)\mathbb{E}[\tau] + (\epsilon E_L + \eta b). \tag{8}$$

where $b = \mathbb{E}[B]$ i.e. mean vacation duration.
Remark: It is valid for any distribution of Off times ($B \sim exp$).

Proposition

The cost $V_e(b)$ is a convex function having a minimum at

$$b_e^\star = \sqrt{\frac{\epsilon E_L \mathbb{E}[\tau]}{\eta}} = \sqrt{\frac{\epsilon E_L \mathbb{E}[\tau]}{\bar{\epsilon} + \epsilon E_S}}. \tag{9}$$

The minimal cost is

$$V_e(b_e^\star) = \epsilon(E_S \mathbb{E}[\tau] + E_L) + 2\sqrt{\epsilon\eta E_L \mathbb{E}[\tau]} \tag{10}$$

Equally Sized B, Hyper-Exponential τ

The total cost

$$V_c(b) = -\bar{\epsilon}\mathbb{E}[\tau] + (\epsilon E_L + \eta b) \sum_{i=1}^{n} \frac{q_i}{1 - \exp(-\lambda_i b)}. \tag{11}$$

where b denotes the vacation size.

Proposition

When $n = 1$, the cost $V_c(b)$ is a convex function having a minimum at

$$b_c^\star = -\frac{1}{\lambda_1}\left(\zeta_1 + W_{-1}(-e^{-\zeta_1})\right) \tag{12}$$

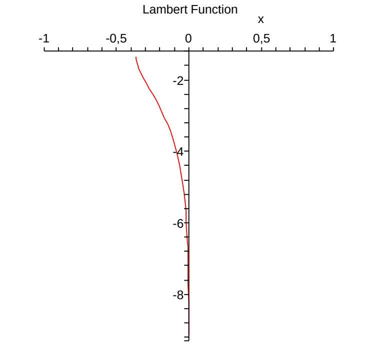
$$\textit{with} \;\; \zeta_1 := \frac{\lambda_1 \epsilon E_L}{\eta} + 1,$$

The minimal cost is

$$V_c(b_c^\star) = -\frac{1}{\lambda}\left(\bar{\epsilon} + \eta W_{-1}(-e^{-\zeta_1})\right). \tag{13}$$

Quick reference -Lambert function W^{-1}

The Lambert W function, satisfies $W(x)\exp(W(x)) = x$.
As the equation $y\exp(y) = x$ has an infinite number of solutions y for each (non-zero) value of x, the function $W(x)$ has an infinite number of branches.
$W_{-1}(-e^x)$ denotes the branch of the Lambert W function that is real-valued on the interval $[-\exp(-1), 0]$ and always below -1.



General Vacations

Scaled General Vacations: $B = \alpha S$.

$$V_s(\alpha) = -\bar{\epsilon}\mathbb{E}[\tau] + (\epsilon E_L + \eta\alpha\mathbb{E}[S]) \sum_{i=1}^{n} \frac{q_i}{1 - \mathcal{S}^*(\alpha\lambda_i)}.$$

The optimization problem can be stated as

$$\min_{\alpha} V_s(\alpha), \quad \text{subject to} \quad \alpha > 0.$$

General Discrete Vacations: $B = \sum_j p_j b_j$

$$V_g(\boldsymbol{p}) = -\bar{\epsilon}\mathbb{E}[\tau] + \sum_{i=1}^{n} \frac{q_i \left(\epsilon E_L + \eta \sum_{j=1}^{J} p_j a_j\right)}{1 - \sum_{j=1}^{J} p_j \exp(-\lambda_i a_j)}.$$

The optimization problem can be stated as

$$\boldsymbol{p}^\star = \arg\min_{\boldsymbol{p}} V_g(\boldsymbol{p}), \ \text{subject to}\, 0 \le p_j \le 1, \ \forall j \ \text{and} \ \sum_{j=1}^{J} p_j = 1. \qquad (14)$$

Distinct Vacation

Vacations Increasing over Time

When $b_k = b_1 f^{\min\{k,l\}}$, and $l := \log_2(b_{max}/b_1)/\log_2 f$.

Optimal Multiplicative Factor:

$$V_m(f) = -\bar{\epsilon}\mathbb{E}[\tau] + \sum_{k=0}^{\infty}\sum_{i=1}^{n} q_i e^{-\lambda_i t_k}\left[\epsilon E_L + \eta b_1 f^{\min\{k,l\}}\right] \quad (15)$$

$$f^\star = \arg\min_{f>1} V_m(f). \quad (16)$$

Remark: $f = 2 \Rightarrow$ IEEE 802.16e type I power saving class strategy.

$$V_{\text{Std}} = -\bar{\epsilon}\,\mathbb{E}[\tau] + \sum_{k=0}^{\infty}\sum_{i=1}^{n} q_i e^{-\lambda_i t_k}\left(\epsilon E_L + \eta b_1 2^{\min\{k,l\}}\right). \quad (17)$$

Dynamic Programming

Recall, one stage cost
$c(\tau(\mathbf{q}), b) = \bar{\epsilon}\mathbb{E}[(b - \tau(\mathbf{q}))\mathbb{1}\{\tau(\mathbf{q}) \le b\}] + \epsilon(E_L + E_S b)$,

DP Equation

$$V(\mathbf{q}) = \min_{b \ge 0} \Big\{ \mathbb{E}[c(\tau(\mathbf{q}), b)] + P(\tau(\mathbf{q}) > b) V(g(\mathbf{q}, b)) \Big\}. \quad (18)$$

where b denotes vacation duration, $\mathbf{q}$ denotes system state, $g(\mathbf{q}, b)$ denote updated state after vacation b.

Starting from $V_0 = 0$, we can use value iteration to compute $V(\mathbf{q})$,

$$V_{k+1}(\mathbf{q}) = \min_{b \ge 0} \Big\{ \mathbb{E}[c(\tau(\mathbf{q}), b)] + P(\tau(\mathbf{q}) > b) V_k(g(\mathbf{q}, b)) \Big\}. \quad (19)$$

Then $V(\mathbf{q}) = \lim_{k \to \infty} V_k(\mathbf{q})$.
Dynamic programming approach facilitates the study of general vacation distribution.

System state

Distribution of residual Off Time τ_t

$$\begin{aligned} P(\tau_t > a) &= P(\tau > t + a \,|\, \tau > t) \\ &= \frac{\sum_{i=1}^{n} q_i \exp(-\lambda_i t) \exp(-\lambda_i a)}{\sum_{j=1}^{n} q_j \exp(-\lambda_j t)} \\ &= \sum_{i=1}^{n} g_i(q_i, t) \exp(-\lambda_i a) \end{aligned} \tag{20}$$

where

$$q_i' = g_i(q_i, t) := \frac{q_i \exp(-\lambda_i t)}{\sum_{j=1}^{n} q_j \exp(-\lambda_j t)}, \quad i = 1, \ldots, n. \tag{21}$$

Remark: Residual time τ_t is also hyper-exponentially distributed.

System state

For hyper-exponential Off time

$g(\mathbf{q}, t)$ is the n-tuple of function $g_i(q_i, t)$,

$$g_i(q_i, t) = \frac{q_i \exp(-\lambda_i t)}{\sum_{j=1}^{n} q_j \exp(\lambda_j t)}$$

Remark: $g(\mathbf{q}, 0) = \mathbf{q}$, $g_i(q_i, b_1 + b_2) = g_i\Big(g_i(q_i, b_1), b_2\Big)$.

Note that the function $g(\mathbf{q}, b)$ updates the state (residual time) after the vacation b.

Lemma : Convergence of state

Fix q and let $I(q)$ be the smallest j for which $q_j > 0$. The following limit holds:

$$\lim_{m \to \infty} g^m(q, T) = e(I(q)).$$

Exponential Off time

- Due to memoryless property residual time τ_t is independent of t, i.e. $\boldsymbol{q}' = \boldsymbol{q}$.
- Single Borel action state space (single parameter λ).

Suggests to have equal sized vacations.

Optimal vacation size

$$V(\mathbf{q}) = \min_{b \geq 0} \left\{ \frac{\mathbb{E}[c(\tau(\mathbf{q}), b)]}{1 - P(\tau(\mathbf{q}) > b)} \right\}.$$

This shows that the optimal vacation is equal sized, unique and is equal to eq. (13).

Hyper Exponential Off Time

Lemma

(i) For all $\mathbf{q}$, $V(\mathbf{q}) \leq \bar{b}$ *where* $\bar{b} = \bar{\epsilon} + \epsilon(1 + \sup_i \frac{1}{\lambda_i})(E_L + E_S)$.
(ii) Without loss of optimality, one may restrict to policies that take only actions within $[0, \tilde{b}]$ *where*

$$\tilde{b} = \frac{\bar{b} + 1 + 1/(\min_i \lambda_i)}{\bar{\epsilon}}$$

$\bar{b}$ corresponds to unit step cost.

General Distribution of Off Time

Proposition

(i) There exists an optimal deterministic stationary policy.
(ii) Let $V^0 := 0$, $V^{k+1} := \mathcal{L}V^k$, *where*

$$\mathcal{L}V(t) := \min_b \{c(t,b) + P(\tau_t > b)V(t+b)\}$$

where $c(t,b)$ *is one stage cost. Then* V^k *converges monotonically to the optimal value* $V^\star$.
(iii) $V^\star$ *is the smallest nonnegative solution of* $V^\star = \mathcal{L}V^\star$. *A stationary policy that chooses at state* t *an action that achieves the minimum of* $\mathcal{L}V^\star$ *is optimal.*

General Distribution of Off Time

Proposition

Assume that τ_t converges in distribution to some limit $\widehat{\tau}$. Define

$$v(b) := \frac{\widehat{c}(b)}{1 - P(\widehat{\tau} > b)}.$$

Then

(i) $\lim_{t\to\infty} V^\star(t) = \min_b v(b)$.

(ii) Assume that there is a unique b that achieves the minimum of $v(b)$ and denote it by $\widehat{b}$. Then there is some stationary optimal policy $b(t)$ such that for all t large enough, $b(t)$ equals $\widehat{b}$.

Suboptimal policies

One stage policy iteration in the class of i.i.d. exponentially distributed vacations (U_1^{exp}) ,

$$V_1^*(\boldsymbol{q}) = \min_{b\geq 0}\Big\{\bar{\epsilon}\mathbb{E}\Big[\Big(b-\tau(\boldsymbol{q})\Big)\mathbb{1}\{\tau(\boldsymbol{q})\leq b\}\Big] + \epsilon(E_L + bE_S) + P\Big(\tau(\boldsymbol{q}) > b\Big)V_e^*(g(\boldsymbol{q},b))\Big\} \quad (22)$$

where $V_e^*(g(\boldsymbol{q},b))$ is equivalent to $V_e^*(b')$, and depends only on the state $g(\boldsymbol{q},b)$; b'^* is obtained from (9).

- Suboptimal policy strictly does better than parametric and easier to compute than Optimal.

Similar approach can be used with deterministic vacations.

Numerical Results

We analyze the sleep mode of IEEE 802.11e using our proposed policies.
Performance metrices

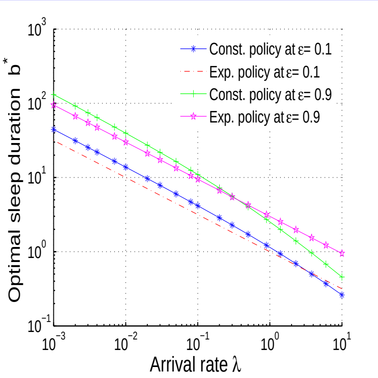
- V^*: captures the energy consumed during the sleep duration and extra delay incurred due to the sleep mode.
- b^*: Optimal mean sleep duration.
- I (Improvement ratio): It quantifies the performance of the policies devised in this paper by looking at the relative improvement with respect to the IEEE 802.16e protocol.

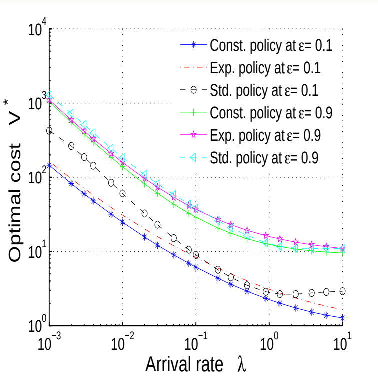
$$I := \frac{V_{\text{Std}} - V_{\text{Optimal}}}{V_{\text{Std}}}.$$

The physical parameters are set to the following values: $E_L = 10$, and $E_S = 1$. The parameters of the Standard are $b_1 = 2$ and $l = 10$.

Exponential Off Time

Impact of λ on optimal and standard policy (IEEE 802.16e protocol).



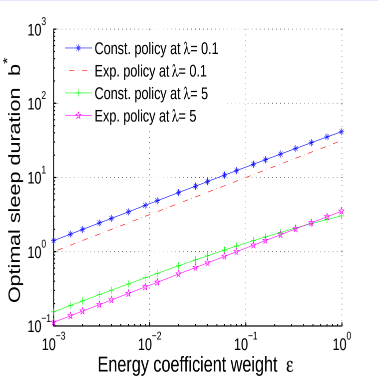


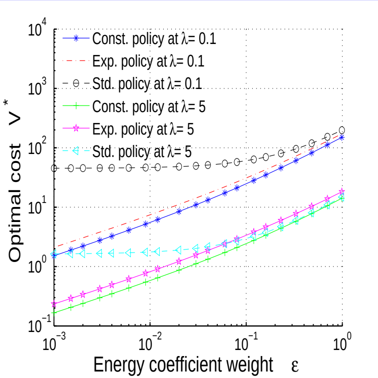
Observations

- Constant policy is the optimal policy.
- Exponential policy is outperformed by standard policy for some λ.

Exponential Off Time

Impact of ϵ on optimal and standard policy.



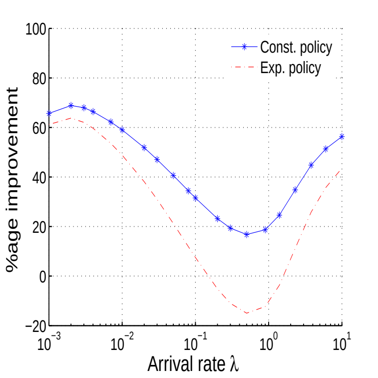


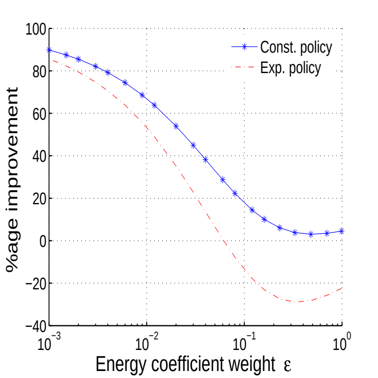
Observations

- Standard policy is fairly insensitive for $\epsilon < 0.1$ (insensitive to delay).
- Exponential policy outperforms the standard policy for some ϵ.

Exponential Off Time

Percentage improvement over standard policy.



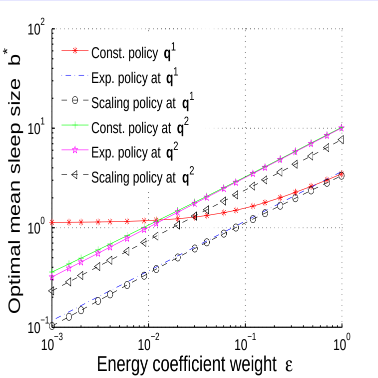


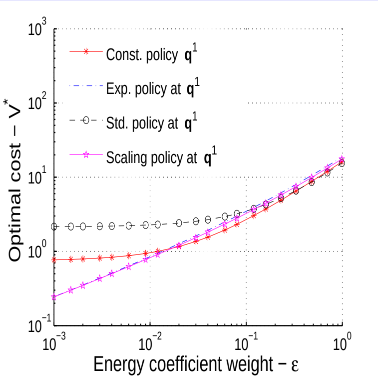
Observations

- Constant policy is always the best policy.
- Exponential policy yields substantial improvement over a large range of values of λ and ϵ.

Hyper Exponential Off Time

Impact of ϵ on optimal and standard policy for hyper-exponential τ at $\lambda = [0.01, 2, 10]$, $\boldsymbol{q}^1 = [0.1, 0.3, 0.6]$, $\boldsymbol{q}^2 = [0.6, 0.3, 0.1]$.



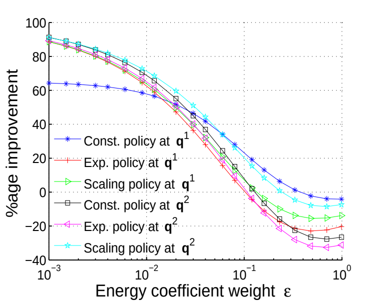


Observations

- For $\epsilon < 0.1$, all proposed policies outperforms standard policy.

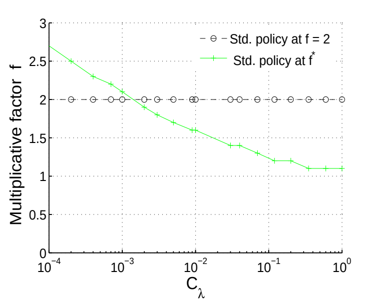
Hyper Exponential Off Time

Percentage improvement over standard policy



$\boldsymbol{q}^1 = [0.1, 0.3, 0.6]$, $\boldsymbol{q}^2 = [0.6, 0.3, 0.1]$

Optimal multiplicative factor for standard policy



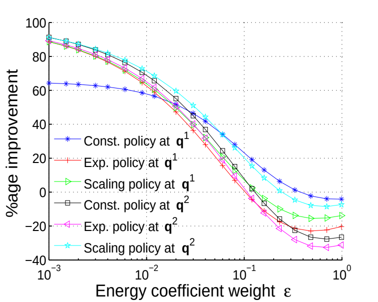
$\boldsymbol{\lambda}_{\text{eff}} = C_\lambda \boldsymbol{\lambda}$, $\boldsymbol{\lambda} = [0.2, 3, 10]$

Observations

- Constant policy outperforms standard policy for $\epsilon \gtrsim 0.4$. No policy is always optimal.
- Optimal multiplicative factor approaches to 1^+ with $C_\lambda \uparrow$.

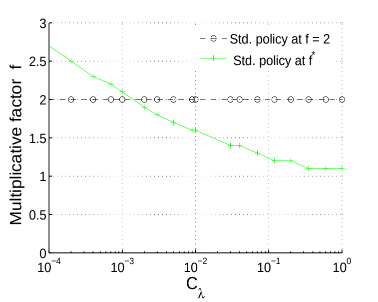
Hyper Exponential Off Time

Percentage improvement over standard policy



$\boldsymbol{q}^1 = [0.1, 0.3, 0.6]$, $\boldsymbol{q}^2 = [0.6, 0.3, 0.1]$

Optimal multiplicative factor for standard policy



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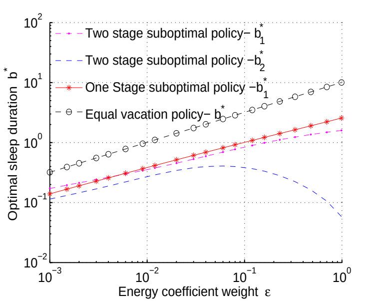
Observations

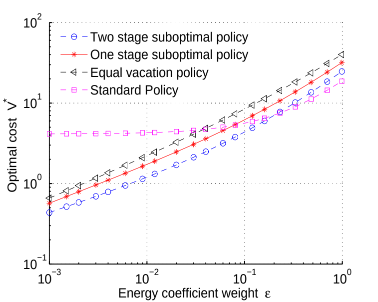
- Constant policy outperforms standard policy for $\epsilon \gtrsim 0.4$. No policy is always optimal.
- Optimal multiplicative factor approaches to 1^+ with $C_\lambda \uparrow$.

Suboptimal Policy

Suboptimal policy (U^1_{Exp}) for hyper-exponential τ at initial distribution

$$\boldsymbol{q} = [0.1, 0.3, 0.6], \ \boldsymbol{\lambda} = [0.2, 3, 10]$$





Observations

- One stage Suboptimal policy is better than Equal vacation.
- At large ϵ system becomes highly delay sensitive, Standard performs better.

Worst Case Performance

when the statistical distribution of the off time is unknown, we optimize the performance under the worst case choice of the unknown parameter.

$$\lambda_w := \arg \max_{\lambda \in [\lambda_a, \lambda_b]} \min_{\{B_k\}, k \in \mathbb{N}^*} V$$

Exponential vacation policy

$$V_e^\star(\lambda) = \epsilon \left(\frac{E_S}{\lambda} + E_L \right) + 2\sqrt{\frac{\epsilon \eta E_L}{\lambda}}.$$

$V_e(b_e^\star)$ is a monotonic function decreasing with λ.
$\lambda_{w,e} = \arg\max_{\lambda \in [\lambda_a, \lambda_b]} V_e^\star(\lambda) = \lambda_a$.
Observe that $\lim_{\lambda \to +\infty} V_e^\star(\lambda) = \epsilon E_L$ and $\lim_{\lambda \to 0} V_e^\star(\lambda) = +\infty$.

Worst Case Performance

Constant vacation policy

$$V_c^\star(\lambda) = \frac{-\bar{\epsilon} - \eta W_{-1}\left(-\exp\left(-1 - \frac{\lambda\epsilon E_L}{\eta}\right)\right)}{\lambda}.$$

$V_c^\star(\lambda)$ is a monotonic function decreasing with λ.
$\lim_{\lambda\to+\infty} V_c^\star(\lambda) = \epsilon E_L$ and $\lim_{\lambda\to 0} V_c^\star(\lambda) = +\infty$.
Evidently, $\lambda_{w,c} = \arg\max_{\lambda\in[\lambda_a,\lambda_b]} V_c^\star(\lambda) = \lambda_a = \lambda_{w,e}$.

We have studied the function using the mathematics software tool, Maple[3] 11.

[3] Maple is a copyright of Maplesoft, a division of Waterloo Maple Inc.

Concluding Remarks

- Introduced a model for control of vacation taking into account the trade off between energy consumption and delays.
- Constant vacation policy is optimal for poisson arrival process.
- Standard protocol is not always optimal even for Hyper-Exponential off time.
- Optimal multiplicative factor asymptotically approaches to 1^+ with increasing arrival rate instead of 2 as proposed by standard protocol.
- No proposed (including standard) policy is always optimal for hyper-exponential. Any adaptive algorithm which can have optimal performance ?

Thanks !!!

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